Rationale (for Monograph on Uniform Convergence)

Uniform Convergence is an important enough topic to deserve a monograph (100 pages or so) if not a book. In the existing literature it tends to get buried in the middle of larger concerns. This makes it somewhat inaccessible and the coverage is usually not comprehensive.

The rationale for devoting a book exclusively to Uniform Convergence is that it is possible in this context to devote time to explaining the concept in detail with plenty of examples. And then to follow up with a wide coverage of applications.

The market for such a publication would be second year undergraduates at British Universities, though one would hope University students abroad might be interested too. There would appear to be a gap in the current literature as far as uniform convergence is concerned. The only available book at the moment which gives a systematic treatment of uniform convergence is Burkill and Burkill which is about to go out of print. This is a gap which needs to be filled fairly urgently particularly when one bears in mind that the new Cambridge Mathematics Syllabus includes uniform convergence and is unable to recommend any other text except Burkill and Burkill at the present time. It is likely that the Cambridge Syllabus will be trend setting and that other universities will adopt a similar syllabus.

Numerous examples (solved in the text) and exercises (to be solved by the reader) are included. Exercises in the body of the text are meant to be straightforward and to illustrate the current point without the complication of some algebraic trick which may hold the student up. More demanding exercises are included at the end of each chapter. Hints and solutions to the exercises will be included at the back of the book, again to avoid the student being held up on side issues.

It is intended to work in the context of piecewise continuous functions. This is sufficient generality to cover all the applications we want to consider. It is necessary to allow discontinuities because of the problem of extending functions periodically when considering their Fourier series. The theory is presented in such a way that it will generalise to Riemann integrable functions easily. Lebesgue integrals are not considered. They represent a higher level of sophistication and are beyond the scope of a book such as this.

Diagrams are drawn separately. The main text (first draft) has been type-set by Plain TeX which does not allow easy inclusion of diagrams. The page numbers given in the Synopsis don't allow for diagrams.

Synopsis (of Monograph on Uniform Convergence)

Chapter 1 Definition of Uniform Convergence Convergence with a parameter. The need for uniform convergence, e.g., for solving O.D.E.'s with power series. Distiction between pointwise convergence and uniform convergence. Definition of uniform convergence of a sequence of functions on an interval by (i) epsilontics, (ii) the M-criterion, (iii) tubular neighbourhoods. (8 pages)

Chapter 2 Analytic Properties of Uniform Limits Uniform limit of a sequence of continuous functions is continuous. Differentiation and integration of sequences of functions term by term. Counter-examples to show exact scope of the results. Almost uniform convergence. (7 pages)

Chapter 3 The Cauchy Criterion Cauchy criterion for numerical sequences and series. Dirichlet and Abel tests for conditional convergence of series. Application to trigonometric series. (8 pages)

Chapter 4 Uniform Convergence of Series Uniform Cauchy Criterion. Uniform absolute convergence. Weierstrass M-test. Test for non-uniform convergence. Uniform Dirichlet and Abel tests. (15 pages)

Chapter 5 Power Series Power series are almost uniformly convergent on their open interval of convergence. Power series can be differentiated and integrated term by term on their open interval of convergence. Maclaurin series, Taylor series. Abel's theorem. Applications to multiplication of series, calculation of π, convergence of the binomial series. (21 pages)

Chapter 6 Fourier Series Uniformly convergent trigonometric series. Fourier coefficients, Fourier series of a piecewise continuous function. Pointwise convergence of piecewise continuously differentiable functions. Uniform convergence of piecewise continuously differentiable functions which are continuous (and whose periodic extensions are continuous). Cesaro summability of Fourier series of continuous functions. Mean square convergence of Fourier series of piecewise continuous functions. (26 pages)

Appendix Divergence of Fourier series of continuous functions. (2 pages)