# An Introduction to Mathematical Analysis

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# Preface

The aim of this book is to give an introduction to that part of mathematics which has come to be known as *analysis*. It is intended to be read by those who have studied calculus from the point of view of its applications, and are now ready for a deeper analysis of the ideas involved. Prerequisites are a working knowledge of the techniques of calculus, and an ability to manipulate logical arguments. The book is envisaged as suitable for students of mathematics in their first year at university.

Two conflicting objectives are present in every area of mathematics. On the one hand there is the desire to understand the underlying principles. On the other hand there is the need to find answers to practical problems. In its early stages a particular piece of mathematics evolves as a tool for solving certain types of problem. Full understanding comes later. Euclid provided the definitive format for proper understanding of an area of mathematics. The fundamental concepts are those of a *theorem* and a *proof*. Since a proof involves logical deduction of one theorem from others it is necessary to start with theorems which are assumed without proof. Such unproved theorems are called *axioms* which are as few and as simple as possible. Having decided what one's axioms are to be, one is then committed to arguing *rigorously* from these axioms and no others. Euclid applied this rationale to geometry. Mathematical analysis is the application of this rationale to the infinitesimal calculus of Newton and Leibnitz.

Since we aim only to introduce analysis, rather than give a formal treatise on the subject, we have made no attempt at complete coverage. We have concentrated on explaining the basic ideas only, having in mind a student who will read this subject among others, who wants quick access to the essentials, and wishes to avoid being held up on abstruse ramifications. At the same time, we hope that those students who intend to specialize in mathematics, and analysis in particular, will find this book an adequate preparation for any later study they may undertake in this area.

A large number of exercises have been included. Those in the body of the text are straightforward and are meant to confirm an idea that has just been introduced, and no more than that. Those at the end of the chapters are more challenging and are meant to flesh out the material of the particular chapter. A few are results which are to be used later, but were considered unsuitable for inclusion in the main text.

There are a few departures from the standard presentation of analysis at this level. Most notable among these are the emphasis on sequential convergence as the definitive limiting process, and the use of sequences to prove the fundamental theorems about continuous functions. The elementary properties of the exponential and trigonometric functions are obtained without calculus. We give a proof of the Riemann integrability of a continuous function which avoids mention of uniform continuity. Continuity and differentiability of power series are proved as special cases of Weierstrassian theorems about infinite series of functions.

I would like to thank Anthony Watkinson for originally inviting me to write the book, Nicholas Browne for comments from the point of view of English sixth formers, Egbert Dettweiler and Brian Hartley for comments derived from using some of the material in teaching undergraduates, Alan Best, David Brannan, and Philip Rippon for discussions about the sequential approach to continuous functions, Beryl Sweeney for her patience and perseverance in typing the manuscript, and finally my wife Suzanne for her unfailing support and encouragement throughout the whole project.

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J. B. R.

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